

This shows that the attenuation constant is sensitive to changes in loss tangent.

The normalized impedance  $z$  in the air-filled portion of waveguide at the interface of the soil sample is equal to the ratio of the propagation constant in the air-filled guide to that in the soil-filled guide and is thus written as

$$z = \frac{\gamma_0}{\gamma} = \left[ \frac{1 - \left( \frac{\lambda_0}{\lambda_c} \right)^2}{\epsilon_r(1 - j \tan \delta) - \left( \frac{\lambda_0}{\lambda_c} \right)^2} \right]^{1/2}. \quad (4)$$

In turn, the VSWR  $S$  in the air-filled guide is related to  $z$  by

$$S = \frac{|z+1| + |z-1|}{|z+1| - |z-1|}. \quad (5)$$

Substitution of (3) and (4) in (5) yields an expression for  $S$  containing  $\epsilon_r$ , the measured quantity  $\alpha/k_0$  and waveguide parameters. For the special case when  $\epsilon_r \geq 2$  and  $\tan \delta \leq 0.1$ , (5) simplifies to

$$S \approx \frac{1}{z} \approx \left[ \frac{\epsilon_r - \left( \frac{\lambda_0}{\lambda_c} \right)^2}{1 - \left( \frac{\lambda_0}{\lambda_c} \right)^2} \right]^{1/2}. \quad (6)$$

This shows that measurement of VSWR is sensitive to changes in dielectric constant.

Eqs. (3), (4) and (5) are plotted in Figs. 3 and 4 as a function of  $\alpha/k_0$  for an operating frequency of 8.6 kMc. Thus, for a given measured  $\alpha/k_0$  and  $S$ , the corresponding dielectric constant may be read from Fig. 3 and then the value of  $\tan \delta$  may be

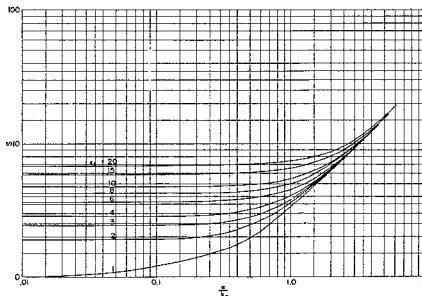


Fig. 3—VSWR as a function of  $\alpha/k_0$  for  $f = 8.6$  kMc.

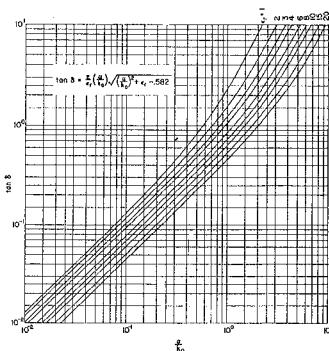


Fig. 4—Loss tangent as a function of  $\alpha/k_0$  for  $f = 8.6$  kMc.

taken from Fig. 4. Not only has this technique proved useful for measuring the properties of soil, but it has also been used with great success in a bio-medical research program for measuring the properties of fatty tissue at  $L$ - and  $S$ -band frequencies.

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and the input signal frequency is in frequency range of oscillation of the amplifier, it is difficult to control the oscillation to obtain stable amplification. On the other hand if the signal frequency is just outside the oscillation region and the impedances and voltages are such that oscillation does not exist, stable amplification is easily obtained.

Thus it can be seen that the output impedance of the amplifier had a great effect on its performance. As an example, with a  $-70$  dbm 70.35 kMc input signal to the VA-99, strong oscillation appeared approximately 30 Mc off the signal. When one of the  $EH$  tuners adjacent to the VA-99 was adjusted, the oscillation weakened as it moved to within 18 Mc of the signal which was now being amplified. By further adjustment of the  $EH$  tuner, oscillation disappeared near the signal frequency and the signal alone remained, amplified 27 db.

When the VA-99 was oscillating strongly

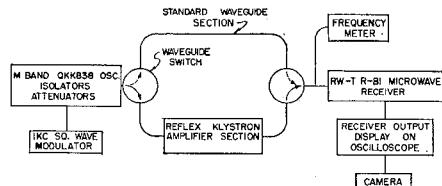


Fig. 1—Test circuit configuration.

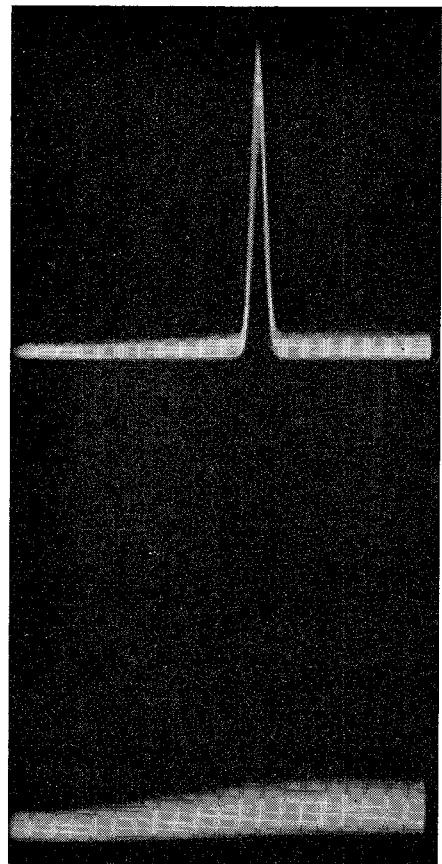


Fig. 2—Pulse amplification (frequency sweep display). Top: Output of VA-99 Amplifier, 30 db gain 70.35 kMc carrier. Bottom: 1-μsec pulse input to VA-99 amplifier, pulse repetition rate, 1000 cps.

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<sup>1</sup> K. Ishii, "X-Band receiving amplifier," *Electronics*, vol. 28, pp. 202-210; April, 1955.

<sup>2</sup> C. F. Quate, R. Kompfner, and D. A. Chisholm, "The reflex klystron as a negative resistance type amplifier," *IRE TRANS. ON ELECTRON DEVICES*, vol. ED-5, pp. 173-179; July, 1958.

<sup>3</sup> K. Ishii, "Using reflex klystron as millimeter wave amplifiers," *Electronics*, vol. 33, pp. 71-73; March, 1960.

<sup>4</sup> D. M. Makurat, R. C. Hertel and K. Ishii, "The reflex klystron as an amplifier at 73 kMc," *PROC. IRE (Correspondence)* vol. 50, pp. 210-211; February, 1962.

<sup>5</sup> K. Ishii, D. E. Schumacher and K. R. Kelly, "Interesting behavior of VA-99 as a millimeter wave amplifier," *PROC. IRE (Correspondence)* vol. 50, p. 2510; December, 1962.

<sup>6</sup> I. Thomas and L. Bounds, "The Investigation of the Characteristics of the KS9-20A Reflex Klystron When Used As an Amplifier," The Mullard Radio Valve Co. Ltd. England, D.V.T. Rept. No. U231; 1961.

it was difficult to control the oscillation merely by the circuit impedance adjustment as Quate, *et al.*,<sup>2</sup> Makurat, *et al.*,<sup>4</sup> and Thomas, *et al.*,<sup>6</sup> experienced. They tried to amplify signals whose frequency was located right at the center of the strong oscillation region of repeller voltage. They loaded the tube down until it stopped oscillating and then placed an input signal to be amplified at the original oscillation frequency. In such cases the amplifiers were often noisy.<sup>2,4,6</sup>

Note that in the method explained in this communication the oscillation frequency was always electrically shifted until the oscillation died out at the signal frequency and the tube was not simply over-loaded to stop oscillation without shifting its frequency.

For small signals (almost the noise level of the RW-T R-B1 receiver 1-μs pulse modulated, (Fig. 2, bottom) the amplifier showed a gain of 30 db at 70.35 kMc (Fig. 2, top).

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### Still Another Method for Transforming Impedances Through Lossless Networks\*

The input impedance  $Z_A$  and the terminating impedance  $Z_L$  of a symmetrical lossless four-terminal network are related by an equation which contains only one additional parameter; namely, the input impedance  $Z_i$  when the network is terminated in any known resistance  $R_0$ . Let  $\Gamma_A = (Z_A - R_0)/(Z_A + R_0)$ ,  $\Gamma_L = (Z_L - R_0)/(Z_L + R_0)$ ,  $\Gamma_i = (Z_i - R_0)/(Z_i + R_0)$ , and the superscript "\*" denote the conjugate. Now

$$\Gamma_A = \frac{\Gamma_i}{\Gamma_i^*} \frac{\Gamma_i^* - \Gamma_L}{1 - \Gamma_i \Gamma_L}. \quad (1)$$

Eq. (1) also relates the voltage reflection coefficients for a lossless two-port junction.<sup>1</sup> This equation provides a basis for a graphical procedure for finding  $Z_A$  or  $Z_L$  when the other is known.

#### SYMMETRICAL NETWORKS

First, it is assumed that the network under consideration is both lossless and symmetrical. Some convenient value of  $R_0$  is

selected and the corresponding value of  $Z_i$  is computed or determined experimentally. (In the case of a two-port junction,  $R_0$  would probably correspond to a matched load.) The impedance  $Z_{1N} = Z_i/R_0$  is plotted on a Smith or Carter chart as shown in Fig. 1. The line  $CZ_{1N}$  is drawn from the center  $C$  of the chart through  $Z_{1N}$ . The point  $M$  is located on this line so that  $(CZ_{1N})(CM) = (\text{radius of chart})^2$ .<sup>2</sup> A graphical procedure for locating  $M$  is to draw the line  $DE$  through  $Z_{1N}$  perpendicular to the line  $CZ_{1N}$ . The points  $D$  and  $E$  are the intersections of this line with the  $R=0$  circle. The lines  $DM$  and  $EM$  are tangent to the  $R=0$  circle.

If  $Z_L$  is known, the impedance  $Z_{LN}^* = Z_L^*/R_0$  is plotted. The line  $GM$  is drawn from  $M$  through  $Z_{LN}^*$ . The line  $Z_{1N}Z_{LN}^*$  is drawn. The line  $CH$  is drawn from  $C$  so that  $\angle Z_{1N}Z_{LN}^* = \angle MCH = \theta$ . The intersection of the lines  $MG$  and  $CH$  is  $Z_{AN} = Z_A/R_0$ .

If  $Z_A$  is known,  $Z_{AN} = Z_A/R_0$  is plotted. The lines  $CZ_{AN}$  and  $MZ_{AN}$  are drawn. The line  $Z_{1N}K$  is drawn from  $Z_{1N}$  so that  $\angle MZ_{AN}C = \angle MZ_{1N}K = \phi$ . The intersection of the lines  $MZ_{AN}$  and  $KZ_{1N}$  is  $Z_{LN}^*$ .

The symmetrical network used for the graphical construction shown in Fig. 1 is the section of lossless transmission line shown in Fig. 2. The characteristic impedance of the line is 100 ohms, its length is 0.15 wavelength,  $R_0 = 50$ ,  $Z_i = 98.21 + j70.05$ ,  $Z_L = 25 + j75$ , and  $Z_A = 17.03 + j27.94$ .

#### TEST FOR LOSSES

Often four-terminal networks and two-port junctions are assumed to be lossless. This can be checked experimentally by

terminating the network in at least two lossless loads, such as a short circuit and an open circuit. If two different input impedances are measured which are pure reactances, then the network is lossless.

#### TEST FOR SYMMETRY

The term *symmetrical network* is used here in the sense that the electrical characteristics of the network are symmetrical at the frequency being used. It is not necessarily symmetrical in appearance or at other frequencies.

A convenient procedure for checking experimentally the symmetry of a lossless network is to terminate the network in a short circuit. Let  $Z_{SC}$  denote the corresponding input impedance. The point  $Z_{SCN} = Z_{SC}/R_0$  is plotted. The network is symmetrical if and only if the point  $Z_{SCN}$  lies on the line from  $Z=0$  to  $M$ . Another test is that

$$X_{SC} = \frac{R_1^2 + X_1^2 - R_0 R_1}{X_1}, \quad (2)$$

where  $Z_{SC} = jX_{SC}$  and  $Z_1 = R_1 + jX_1$ , if and only if the network is symmetrical.

For the network shown in Fig. 2,  $Z_{SC} = j137.64$  and  $Z_{SCN}$  lies on the line from  $Z=0$  to  $M$ , as shown in Fig. 1. The network in Fig. 2 is made unsymmetrical by the addition of the shunt reactance  $X_2 = 200$  as shown in Fig. 3 (next page). The value of  $Z_{SCN}$  remains unchanged and the new value of  $Z_1$  is  $121.4 + j84.44$ . The point  $Z_{SCN}$  does not lie on the line from  $Z=0$  to  $M$ , as shown in Fig. 4. If  $Z_{SC} = 0$ , then  $Z_{LN}$  must lie on the circle shown in Fig. 5 when the network is symmetrical.

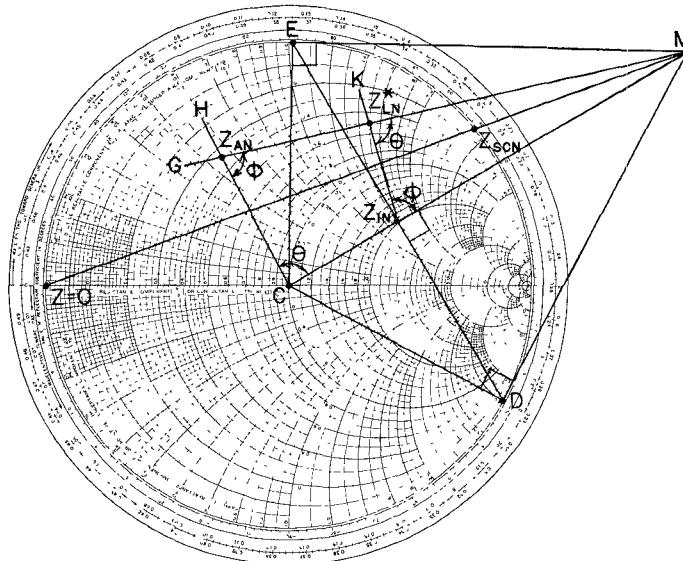


Fig. 1—Graphical construction for a symmetrical network.

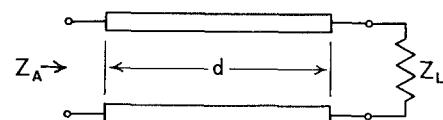


Fig. 2—A symmetrical network.

\* Received January 14, 1963.

<sup>1</sup> D. Kajfez, "Wide-band matching of lossless waveguide two-ports," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 174-178; May, 1962.